2.6 Prove Statements about Segments and Angles

Before You used deductive reasoning.

Now You will write proofs using geometric theorems.

Why? So you can prove angles are congruent, as in Ex. 21.

Key Vocabulary
- **proof**
- **two-column proof**
- **theorem**

A **proof** is a logical argument that shows a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

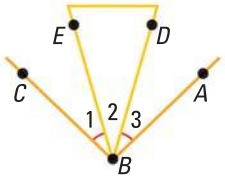
EXAMPLE 1 Write a two-column proof

WRITE PROOFS
Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

Write a two-column proof for the situation in Example 4 on page 107.

GIVEN ▶ $m\angle 1 = m\angle 3$

PROVE ▶ $m\angle EBA = m\angle DBC$



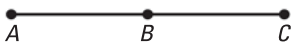
STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle EBA = m\angle 3 + m\angle 2$	2. Angle Addition Postulate
3. $m\angle EBA = m\angle 1 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle DBC$	4. Angle Addition Postulate
5. $m\angle EBA = m\angle DBC$	5. Transitive Property of Equality

✓ GUIDED PRACTICE for Example 1

1. Four steps of a proof are shown. Give the reasons for the last two steps.

GIVEN ▶ $AC = AB + AB$

PROVE ▶ $AB = BC$



STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. Segment Addition Postulate
3. $AB + AB = AB + BC$	3. ?
4. $AB = BC$	4. ?

THEOREMS The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

TAKE NOTES

Be sure to copy all new theorems in your notebook. Notice that the theorem box tells you where to find the proof(s).

THEOREMS *For Your Notebook*

THEOREM 2.1 Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs: p. 137; Ex. 5, p. 121; Ex. 26, p. 118

THEOREM 2.2 Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs: Ex. 25, p. 118; Concept Summary, p. 114; Ex. 21, p. 137

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

a. If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$.

b. If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$.

Solution

a. Transitive Property of Angle Congruence

b. Symmetric Property of Segment Congruence

GUIDED PRACTICE for Example 2

Name the property illustrated by the statement.

2. $\overline{CD} \cong \overline{CD}$

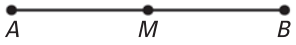
3. If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$.

In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs so that you will be prepared to write more complicated proofs in later chapters.

EXAMPLE 3 Use properties of equality

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.

WRITE PROOFS
Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.



GIVEN ▶ M is the midpoint of $\overline{AB}$.

PROVE ▶ **a.** $AB = 2 \cdot AM$

b. $AM = \frac{1}{2}AB$

STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of midpoint
3. $AM = MB$	3. Definition of congruent segments
4. $AM + MB = AB$	4. Segment Addition Postulate
5. $AM + AM = AB$	5. Substitution Property of Equality
a. 6. $2AM = AB$	6. Distributive Property
b. 7. $AM = \frac{1}{2}AB$	7. Division Property of Equality

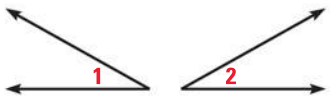
GUIDED PRACTICE for Example 3

4. **WHAT IF?** Look back at Example 3. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2}AB$ instead?

CONCEPT SUMMARY *For Your Notebook*

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs.

Proof of the Symmetric Property of Angle Congruence

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Statements based on facts that you know or on conclusions from deductive reasoning

Definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

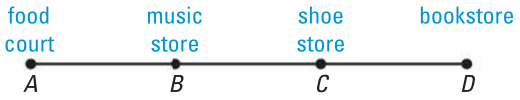
EXAMPLE 4 Solve a multi-step problem

SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 120 for the **Problem Solving Workshop**.

Solution

STEP 1 **Draw** and label a diagram.



STEP 2 **Draw** separate diagrams to show mathematical relationships.

STEP 3 **State** what is given and what is to be proved for the situation. Then write a proof.

GIVEN ▶ B is the midpoint of $\overline{AC}$.
C is the midpoint of $\overline{BD}$.

PROVE ▶ $AB = CD$

STATEMENTS	REASONS
1. B is the midpoint of $\overline{AC}$. C is the midpoint of $\overline{BD}$.	1. Given
2. $\overline{AB} \cong \overline{BC}$	2. Definition of midpoint
3. $\overline{BC} \cong \overline{CD}$	3. Definition of midpoint
4. $\overline{AB} \cong \overline{CD}$	4. Transitive Property of Congruence
5. $AB = CD$	5. Definition of congruent segments

GUIDED PRACTICE for Example 4

5. In Example 4, does it matter what the actual distances are in order to prove the relationship between AB and CD? *Explain*.

6. In Example 4, there is a clothing store halfway between the music store and the shoe store. What other two store entrances are the same distance from the entrance of the clothing store?

2.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 4, 12, 19, 27, and 28

SKILL PRACTICE

1. **VOCABULARY** What is a *theorem*? How is it different from a *postulate*?

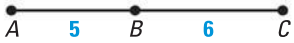
2. ★ **WRITING** You can use theorems as reasons in a two-column proof. What other types of statements can you use as reasons in a two-column proof? Give examples.

EXAMPLE 1 on p. 112 for Exs. 3–4

3. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN ▶ $AB = 5$, $BC = 6$

PROVE ▶ $AC = 11$



STATEMENTS	REASONS
1. $AB = 5$, $BC = 6$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = 5 + 6$	3. ?
4. ?	4. Simplify.

4. ★ **MULTIPLE CHOICE** Which property listed is the reason for the last step in the proof?

GIVEN ▶ $m\angle 1 = 59°$, $m\angle 2 = 59°$

PROVE ▶ $m\angle 1 = m\angle 2$

STATEMENTS	REASONS
1. $m\angle 1 = 59°$, $m\angle 2 = 59°$	1. Given
2. $59° = m\angle 2$	2. Symmetric Property of Equality
3. $m\angle 1 = m\angle 2$	3. ?

Ⓐ Transitive Property of Equality Ⓑ Reflexive Property of Equality

Ⓒ Symmetric Property of Equality Ⓓ Distributive Property

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 5–13

USING PROPERTIES **Use the property to copy and complete the statement.**

5. Reflexive Property of Congruence: ? $\cong \overline{SE}$

6. Symmetric Property of Congruence: If ? $\cong$?, then $\angle RST \cong \angle JKL$.

7. Transitive Property of Congruence: If $\angle F \cong \angle J$ and ? $\cong$?, then $\angle F \cong \angle L$.

NAMING PROPERTIES **Name the property illustrated by the statement.**

8. If $\overline{DG} \cong \overline{CT}$, then $\overline{CT} \cong \overline{DG}$.

9. $\angle VWX \cong \angle VWX$

10. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{XY}$, then $\overline{JK} \cong \overline{XY}$.

11. $YZ = ZY$

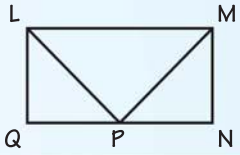
12. ★ **MULTIPLE CHOICE** Name the property illustrated by the statement "If $\overline{CD} \cong \overline{MN}$, then $\overline{MN} \cong \overline{CD}$."

Ⓐ Reflexive Property of Equality Ⓑ Symmetric Property of Equality

Ⓒ Symmetric Property of Congruence Ⓓ Transitive Property of Congruence

13. ERROR ANALYSIS In the diagram below, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. *Describe* and correct the error in the reasoning.

Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Reflexive Property of Segment Congruence.



EXAMPLE 4 on p. 115 for Exs. 14–15

MAKING A SKETCH In Exercises 14 and 15, sketch a diagram that represents the given information.

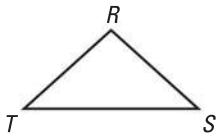
14. CRYSTALS The shape of a crystal can be represented by intersecting lines and planes. Suppose a crystal is *cubic*, which means it can be represented by six planes that intersect at right angles.

15. BEACH VACATION You are on vacation at the beach. Along the boardwalk, the bike rentals are halfway between your cottage and the kite shop. The snack shop is halfway between your cottage and the bike rentals. The arcade is halfway between the bike rentals and the kite shop.

16. DEVELOPING PROOF Copy and complete the proof.

GIVEN ▶ $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$

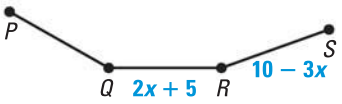
PROVE ▶ $\overline{RS} \cong \overline{TS}$



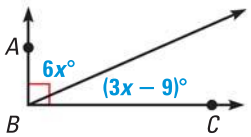
STATEMENTS	REASONS
1. $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$	1. ?
2. $RS = RT$	2. Transitive Property of Equality
3. $RT = TS$	3. Definition of congruent segments
4. $RS = TS$	4. Transitive Property of Equality
5. $\overline{RS} \cong \overline{TS}$	5. ?

ALGEBRA Solve for x using the given information. *Explain* your steps.

17. GIVEN ▶ $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$



18. GIVEN ▶ $m\angle ABC = 90°$



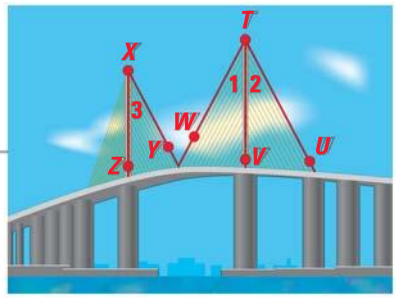
19. ★ SHORT RESPONSE *Explain* why writing a proof is an example of deductive reasoning, not inductive reasoning.

20. CHALLENGE Point P is the midpoint of $\overline{MN}$ and point Q is the midpoint of $\overline{MP}$. Suppose $\overline{AB}$ is congruent to $\overline{MP}$, and $\overline{PN}$ has length x. Write the length of the segments in terms of x. *Explain*.

a. $\overline{AB}$ **b.** $\overline{MN}$ **c.** $\overline{MQ}$ **d.** $\overline{NQ}$

PROBLEM SOLVING

21. BRIDGE In the bridge in the illustration, it is known that $\angle 2 \cong \angle 3$ and $\overrightarrow{TV}$ bisects $\angle UTW$. Copy and complete the proof to show that $\angle 1 \cong \angle 3$.



STATEMENTS	REASONS
1. $\overrightarrow{TV}$ bisects $\angle UTW$.	1. Given
2. $\angle 1 \cong \angle 2$	2. ?
3. $\angle 2 \cong \angle 3$	3. Given
4. $\angle 1 \cong \angle 3$	4. ?

@HomeTutor for problem solving help at classzone.com

EXAMPLE 3 on p. 114 for Ex. 22

22. DEVELOPING PROOF Write a complete proof by matching each statement with its corresponding reason.

GIVEN ▶ $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$.

PROVE ▶ $m\angle PQS = \frac{1}{2} m\angle PQR$

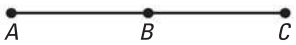
STATEMENTS	REASONS
1. $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$.	A. Definition of angle bisector
2. $\angle PQS \cong \angle SQR$	B. Distributive Property
3. $m\angle PQS = m\angle SQR$	C. Angle Addition Postulate
4. $m\angle PQS + m\angle SQR = m\angle PQR$	D. Given
5. $m\angle PQS + m\angle PQS = m\angle PQR$	E. Division Property of Equality
6. $2 \cdot m\angle PQS = m\angle PQR$	F. Definition of congruent angles
7. $m\angle PQS = \frac{1}{2} m\angle PQR$	G. Substitution Property of Equality

@HomeTutor for problem solving help at classzone.com

PROOF **Use the given information and the diagram to prove the statement.**

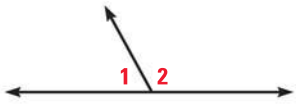
23. GIVEN ▶ $2AB = AC$

PROVE ▶ $AB = BC$



24. GIVEN ▶ $m\angle 1 + m\angle 2 = 180°$, $m\angle 1 = 62°$

PROVE ▶ $m\angle 2 = 118°$

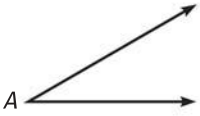


PROVING PROPERTIES **Prove the indicated property of congruence.**

25. Reflexive Property of Angle Congruence

GIVEN ▶ A is an angle.

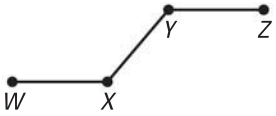
PROVE ▶ $\angle A \cong \angle A$



26. Transitive Property of Segment Congruence

GIVEN ▶ $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$

PROVE ▶ $\overline{WX} \cong \overline{YZ}$



◯ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

27. ★ **SHORT RESPONSE** In the sculpture shown, $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$. Classify the triangle and *justify* your reasoning.

28. ★ **SHORT RESPONSE** You use a computer drawing program to create a line segment. You copy the segment and paste it. You copy the pasted segment and then paste it, and so on. How do you know all the line segments are congruent?

EXAMPLE 4 on p. 115 for Ex. 29

29. **MULTI-STEP PROBLEM** The distance from the restaurant to the shoe store is the same as the distance from the cafe to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the cafe, and from the florist to the dry cleaners.

Use the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the cafe to the dry cleaners.

a. Draw and label a diagram to show the mathematical relationships.

b. State what is given and what is to be proved for the situation.

c. Write a two-column proof.

Animated Geometry at classzone.com

30. **CHALLENGE** The distance from Springfield to Lakewood City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville is. Moon Valley is 50 miles farther from Springfield than Lakewood City is.

a. Assume all five cities lie in a straight line. Draw a diagram that represents this situation.

b. Suppose you do not know that all five cities lie in a straight line. Draw a diagram that is different from the one in part (a) to represent the situation.

c. *Explain* the differences in the two diagrams.

MIXED REVIEW

PREVIEW Prepare for Lesson 2.7 in Exs. 31–33.

Given $m\angle 1$, find the measure of an angle that is complementary to $\angle 1$ and the measure of an angle that is supplementary to $\angle 1$. *(p. 35)*

31. $m\angle 1 = 47°$ **32.** $m\angle 1 = 29°$ **33.** $m\angle 1 = 89°$

Solve the equation. Write a reason for each step. *(p. 105)*

34. $5x + 14 = -16$ **35.** $2x - 9 = 15 - 4x$ **36.** $x + 28 = -11 - 3x - 17$